D. Layzer: Cosmogenesis

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The Importance of Being Discrete

The world of Galileo, Newton, and Maxwell—the world of classical physics—is fundamentally continuous. In Chapters 2 and 3, we saw that Boltzmann's definition of randomness and the Strong Cosmological Principle, the twin pillars supporting the theory of timebound order sketched there, depend on the assumption that the world is fundamentally discrete. In this chapter and the next, we will try to gain insight into what is arguably the most important scientific discovery of the twentieth century, the discovery that physical reality is at bottom discrete.

"The Point at Which Science Must Stop"

Newton's *Mathematical Principles of Natural Philosophy* (the *Principia*) appeared in 1687. During the following century, scientists devoted most of their efforts to working out its implications. They constructed mathematical theories that described the motions of particles, of solid bodies, of fluids, and of the planets and their satellites, all based on Newton's three laws of motion and his theory of universal gravitation. With these branches of science well secured, nineteenth-century scientists set out to conquer new territories. Experimental and theoretical investigations of light, electricity, and magnetism culminated in James Clerk Maxwell's electrodynamics, a theory that expanded the framework of Newtonian physics and brought all three groups of phenomena under a common roof. Studies of heat culminated in the science of thermodynamics, with its two great laws: the law of conservation of energy (formulated by Robert Mayer and James Joule) and the law of entropy growth (formulated by Rudolf Clausius). Unlike electromagne-

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tism, however, thermodynamics had only a tenuous link with Newtonian physics, through the concept of energy. Maxwell, Clausius, and Ludwig Boltzmann immediately set out to construct a dynamical basis for the science of heat. Their point of departure was the atomic hypothesis. Their aim was to show that the macroscopic properties of gases, liquids, and solids could be deduced from the properties and behavior of their constituent molecules, moving and interacting according to Newton's laws of motion. In its initial stages, this program was spectacularly successful. By 1880, many scientists had come to believe that all the laws of physics had been discovered and that the remaining unsolved problems—the problem of radiation, for example—wouldn't hold out much longer.

Not all physicists were so sanguine, however. In a popular lecture published in 1880, Maxwell began by explaining why he and other physicists were convinced that gases were made up of molecules moving and interacting according to the laws of Newtonian physics. But, he went on, molecules can't be just miniature billiard balls:

> The molecule, though indestructible, is not a hard rigid body, but is capable of internal movements, and when these are excited, it emits rays, the wave-length of which is a measure of the time of vibration of the molecule.

Less concisely: if we look at a glowing cloud of gas through a spectroscope, whose prism spreads the light into a rainbow, or spectrum, we see a set of bright lines, each a definite pure color. A sodium lamp, for example, emits most of its visible light in two closely spaced yellow lines. The colors (or wavelengths) of these lines are characteristic of the molecules that make up the cloud. Every atom and molecule has its own pattern of bright lines, as distinctive as a thumbprint. What is more—and this is the crux of Maxwell's argument—careful measurements have shown that the patterns of lines emitted by samples of atomic hydrogen obtained from different terrestrial sources are identical with one another and with the patterns imprinted on the light we receive from the Sun, the stars, and interstellar gas clouds.

> We are thus assured that molecules [we would now call them atoms] of the same nature as those of our hydrogen exist in those distant regions, or at least did exist when the light by which we see them was emitted. . . . A molecule of hydrogen, . . . whether in Sirius or Arcturus, executes its vibrations in precisely the same time.

It follows, said Maxwell, that the vibrations in a hydrogen "molecule" don't obey Newton's laws. This becomes obvious when we compare them with the motions of planets in the solar system, which we know obey Newton's laws:

> The form and dimensions of the orbits of the planets, for instance, are not determined by any law of nature, but depend upon [initial conditions]. . . . [N]atural causes . . . are at work, which tend to modify, if they do not at length destroy, all the arrangements of the earth and the whole solar system. But though in the course of ages catastrophes have occurred and may yet occur in the heavens,

though ancient systems may be dissolved and new systems evolved out of their ruins, the molecules out of which these systems are built—the foundation stones of the material universe—remain unbroken and unworn.

Maxwell concluded that physics had reached an impasse:

> We have been led, along a strictly scientific path, very near to the point at which Science must stop.[1]

By "Science" Maxwell meant not just the physics of his own day but any theory of the kind that modern physicists call "classical." A classical theory pictures the physical world as made up of *events*, each referring to a definite point in space and a definite instant of time. The mathematical laws of a classical theory link events that refer to a given moment of time with events that refer to earlier and later moments.

What are these events? Newton, like Democritus, pictured the world as made up of particles moving in empty space. In Newtonian physics, every event is associated with a particle and is defined by the particle's position, velocity, and mass. Around the middle of the nineteenth century, Michael Faraday added an important new element to this picture, the electromagnetic field. Faraday thought of electric and magnetic fields as permeating the space between electrically charged particles, the sources of the field. The electric field at any given point in space is equal to the force that would be experienced by a stationary particle at that point carrying one unit of positive electric charge, but the field is there whether or not a charged particle is there to experience it. Analogously, a magnetic field may be defined by its action on a piece of wire carrying an electric current or on a moving electric charge.

Although the introduction of the electromagnetic field greatly enriched classical physics, it didn't alter its basic character. Neither did Einstein's two great theories of space, time, and energy: special relativity (1905) and general relativity (1915); they continued to describe the world in a "classical" way—that is, as consisting of events in a spacetime continuum, albeit a spacetime continuum with radically different properties from those attributed to it by earlier scientists.

The Quantum World

Maxwell was perfectly correct in his belief that science, in the only sense that would have been intelligible to a scientist of the late nineteenth century, would never be able to explain why the "foundation stones of the material universe" are forever "unbroken and unworn." Quantum physics, which emerged during the first quarter of the twentieth century, *did* explain this, and much more besides. But quantum physics is not a classical theory. Its picture of physical reality differs so radically from that of classical physics that the term *picture* has to be stretched to apply to it. How do you picture a pointlike particle that can be in many places at the same time? According to quantum physics, the electron is such a particle.

Or rather, that is the kind of statement about electrons you get when you try to translate the language of quantum physics into English. The result doesn't make sense—and certainly can't be illustrated by a picture.

The basic concepts of classical physics are not far removed from experience and ordinary language. (At least they don't seem so today.) Words like *velocity, acceleration, mass,* and *force* may not mean quite the same things to a poet as to a physicist. Still, the differences in meaning aren't likely to give rise to gross misunderstandings. The basic concepts of quantum physics refer to mathematical notions that are far less directly connected with experience. Yet the quantum world does have a coherent and understandable structure, some of whose most important features can, I believe, be communicated in largely nonmathematical language.

The quantum world differs from the classical world in four important ways.

1. The classical world is continuous; the quantum world is discrete. The terms *continuous* and *discrete* apply to collections. The members of a discrete collection are separate from one other; each one has some space, real or metaphorical, around it. In a continuous collection, or continuum, there are no gaps. The points on a line form a continuum; the divisions on a ruler form a discrete set.

Maxwell's comparison between atoms and planetary systems illustrates this contrast between the classical and quantum worlds. An artificial satellite has a continuous range of possible periods and orbital diameters, but a hydrogen atom must have a discrete set of possible physical states, because it emits and absorbs light only at particular wavelengths. The chemical elements themselves, arranged in order of mass or of the electric charge carried by the atomic nucleus, form a discrete sequence. There is no element whose mass lies between the mass of hydrogen (1 unit) and the mass of deuterium (2 units), or whose nuclear electric charge lies between those of hydrogen (1 unit) and helium (2 units). In classical physics, mass and electric charge have continuous ranges of possible values.

2. The classical world is a world of flux and irreversible change. But crystals, molecules, atoms, and subatomic particles—the objects that populate the quantum world—are permanent and immutable.

3. In the classical world, change is both continuous and, in principle, completely predictable. The observable effects of change in the quantum world are discontinuous and, in principle, partly unpredictable. When a hydrogen atom is observed to emit a photon, its physical state changes abruptly. According to quantum physics, it is impossible to predict exactly when such an event will occur. Theory tells us only what fraction of the atoms in a cloud of excited hydrogen atoms will emit a photon during a given interval of time.

4. The classical world is populated by individuals; the quantum world is populated by clones. Two classical objects—a pair of ball bearings, for example—can't be precisely alike in every respect. But according to quantum physics, all electrons are exact replicas of one another. They are indistinguishable not only in practice but also in principle, as are all hydrogen atoms, all water molecules, and all salt crystals (apart from size). This is not simply a dogma, but a testable and strongly corroborated hypothesis. For example, if electrons weren't absolutely indistinguishable, two hydrogen atoms would form a much more weakly bound

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molecule than they actually do. The absolute indistinguishability of the electrons in the two atoms gives rise to an "extra" attractive force between them. The indistinguishability of electrons is also responsible for the structure of the periodic table—that is, for the fact that elements in the same column of the table (inert gases, halogens, alkali metals, alkali earths, and so on) have similar chemical properties.

These four nonclassical aspects of the quantum world are closely related. Perhaps the most fundamental of them is discreteness, because it is implicated in all the others. For example, consider the second property, permanence. The fact that atoms and molecules have fixed properties depends on the fact that their possible physical states are discrete rather than continuous. When the physical state of an atom or a molecule changes, it changes from one well-defined state to another.

Indistinguishability, the fourth property, also depends on discreteness. Electrons wouldn't be indistinguishable if their electric charge could assume a continuous range of possible values.

Finally, discreteness entails unpredictability, the third property. This connection is a little subtler than the others. Consider a collection of free neutrons. Outside the atomic nucleus, the neutron is an unstable particle, decaying spontaneously into a proton, an electron, and an antineutrino (a massless particle that interacts very weakly with other kinds of matter). After about ten and a half minutes, half the neutrons in a large sample will have decayed. Quantum physics doesn't, however, predict when an individual neutron will decay. Now suppose that neutrons had some as-yet-unknown property that, if it could be observed, would enable us to predict their individual lifetimes. Since the observed lifetimes of individual neutrons form a continuum, the possible values of this hypothetical property would also have to form a continuum. But if the internal states of atomic and subatomic objects form discrete aggregates, this is impossible. Thus discreteness is incompatible with the predictability of certain kinds of atomic events.

Discreteness and Continuity in Greek Mathematics

The first generations of Greek mathematicians recognized that discreteness and continuity are complementary and contradictory aspects of the world of mathematics (which they believed underlies the world of experience). Arithmetic is the realm of the discrete; geometry, of the continuous. We can represent the integers by line segments whose lengths are multiples of an arbitrarily chosen unit segment, but we can't always represent line segments by integers. For example, it isn't possible to represent a side and a diagonal of a square simultaneously by integers. Pythagoras, or one of his disciples, discovered this by the following argument:

Suppose it *was* possible to represent a side and a diagonal of a square by two integers, s and d, so that the side contained s appropriately chosen units of length and the diagonal, d units. We may assume that s and d have no common divisor; if they did have a common divisor, c, we could choose a new unit of length c

times as long as the original one, and then the square's side and diagonal would be represented by integers without a common divisor. Pythagoras's theorem tells us that $d^2 = 2s^2$. But that is impossible, because it implies that d is an even number, which implies that d^2 is divisible by 4, which, in turn, implies that s^2 is even, and hence that s itself is even, contrary to our assumption that s and d have no common divisor. So our original premise, that there exists a common unit of measurement for a side and a diagonal of a square, must be false.

We may think of geometry, with its continuously variable figures and magnitudes, as a metaphor for the world of everyday experience, and of arithmetic, the science of integers, as a metaphor for the discrete, unchanging quantum world that underlies experience. But the relations between geometry and classical physics and between arithmetic and quantum physics are more than metaphorical. Classical physics is a direct descendant of Greek geometry. Archimedes widened the net of Euclid's geometry by adding axioms about weight, force, and the balance of forces. Archimedean statics forms the basis of the modern mechanical engineering curriculum. Archimedes himself invented many practical applications of his theories, among them a variety of devices for redirecting and multiplying forces, such as pulleys, winches, and the hydraulic screw. Galileo began where Archimedes had left off. His earliest efforts were the solutions of problems that Archimedes might have assigned to a graduate student, had he had one. Galileo went on, however, to extend Greek mathematical physics in a direction that no Greek, not even Archimedes, had anticipated: the description of motion—the motion of bodies sliding without friction down inclined planes and curved surfaces, falling from towers, being shot out of cannons. Finally, Galileo's new science of motion, along with Johannes Kepler's theory of planetary motion, itself a direct continuation of the work of a long line of Greek mathematicians, paved the way for the central achievement of classical physics: Newton's theory of motion and universal gravitation.

Quantum physics has no such direct link with the Greek past. Yet its worldview has much in common with that of Pythagoras and his followers, who believed that mathematical objects and relations are the building blocks of physical reality. Aristotle, who was not well disposed toward this worldview—he considered mathematics to be an idealized representation of the surface appearance of things—tells us in the *Metaphysics* that the Pythagoreans "supposed the whole heaven to be a *harmonia* and a number." The Pythagorean notion of harmony will repay a closer look because, as we will see, it also figures in the quantum physicist's picture of physical reality—not, of course, in a way that the Pythagoreans could have anticipated, but in a way they surely would have appreciated.

Pythagoras's Theory of Musical Harmony

Pythagoras discovered that consonant musical intervals—in his day they would have been octaves, fifths, and fourths—are produced by vibrating strings that have the same thickness, density, and tension and whose lengths are in the ratio of

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small integers. Halving the length of a vibrating string raises its pitch an octave. Reducing its length by a third raises its pitch a fifth. Thus octaves correspond to the ratio 2/1; fifths, to the ratio 3/2. The interval of a fourth is a fifth below the octave, so it corresponds to the ratio $2/1 \times 2/3 = 4/3$. Moving upward and downward in fifths from an arbitrary starting point (middle C, say) and calling notes that differ by an octave by the same name, we obtain a sequence that includes

. . . , E-flat, B-flat, F, C, G, D, A, E, B, F-sharp, C-sharp, G-sharp, . . .

Each set of seven consecutive notes in this sequence yields a diatonic scale, the scale defined by seven consecutive white keys on a piano. For example, the sequence beginning with F contains the seven tones of the C major scale; the sequence beginning with E-flat contains the seven tones of the B-flat major scale; and so on. Each set of five consecutive notes yields a pentatonic scale, the scale defined by five consecutive black keys on a piano. The pentatonic scale is common in folk music.

Pythagoras's scale isn't quite identical with the modern one, though. A sequence of perfect fifths goes on endlessly in both directions. E-sharp is a little sharper than F; C-flat is a little flatter than B; and so on. But Pythagoras's theory suggests a way to close up the sequence, to make it into a *circle* of fifths containing twelve and only twelve distinct tones. The theory makes two distinct assertions: (1) that equal musical intervals correspond to equal ratios of the lengths of vibrating strings of given density, thickness, and tension; and (2) that *consonant* intervals correspond to ratios between small whole numbers. To close the circle of fifths, we have to divide the octave into twelve equal intervals. Pythagoras's first assertion implies that the ratio corresponding to the interval between successive tones in a scale containing twelve equal intervals must be the twelfth root of 2. Since the fifth tone of the diatonic scale (G in the C major scale) is the seventh tone of the twelve-tone scale, the interval of a fifth must correspond to the seven-twelfths power of 2, or 1.498307. This differs from a perfect Pythagorean fifth, corresponding to the ratio 1.5, by slightly more than one part in a thousand.

In one of his Norton Lectures, on music of the golden age from Bach to Beethoven, Leonard Bernstein refers to the interplay between the "two forces of chromaticism and diatonicism, forces that were equally powerful and presumably contradictory in nature. This point of delicate balance is like the still center in the flux of musical history." The preceding calculation shows that this delicate balance is a consequence of the fact that the number 2 raised to the seven-twelfths power is very close to 3/2.

Nowadays we regard music and physics as being at opposite poles of the cultural spectrum, but to Pythagoras and his contemporaries the properties of musical sounds were no less a part of the natural world than the properties of solids. The rule that every musical interval corresponds to a definite ratio between the lengths of vibrating strings of the same density, thickness, and tension must have seemed to them just as objective as the rule that the volumes of similar solids are proportional to the cubes of corresponding edges: both would have been regarded as natural laws. And the connection between *consonant* musical intervals and whole numbers might easily have been seen as the tip of an iceberg, affording the first glimpse of a deep correspondence between natural order and mathematical order.

Although the Pythagorean belief in an underlying mathematical order is shared by most contemporary theoretical physicists, some historians and philosophers of science dismiss it as a form of mysticism akin to numerology and astrology. The difference between the Pythagoreans and modern physicists, according to these writers, is that modern physicists, although they may be mystics at heart, submit the laws they devise to rigorous testing, whereas the ancient Greeks, as everybody knows, were long on speculation but short on testing. The evidence usually adduced to support this view is that Plato consistently disparaged experience as a source of knowledge.

Plato wasn't a practicing scientist, however. Even though documentary evidence is lacking, I think we can safely assume that experiments and observations played important roles in the development of all branches of Greek mathematical science: geometry, harmony, optics, astronomy, mechanics, and hydrostatics. Why? Because Greek science is highly veridical. The theorems of Euclidean geometry express verifiable properties of rigid bodies. The theorems of Greek optics correctly describe how light-rays are reflected from plane and spherical mirrors. Pythagoras's theory of musical harmony works. So does Aristarchus's heliocentric model of the Solar System, which Kepler took as his starting point. And so does Archimedes's theory of statics, which Galileo, Huygens, and Newton took as theirs. Unless we assume that Greek mathematical scientists were clairvoyant, we must assume that in formulating their theories they relied heavily on experiments and observations, although, of course, their experiments may have been less systematic than those of seventeenth-century scientists.

Overtones

To understand the connection between musical harmony and quantum physics, we have to dig a little deeper into the question of why certain intervals, such as octaves and fifths, are pleasing to the ear, while others—octaves and fifths played slightly out of tune, for instance—are not. The answer (insofar as it falls within the domain of physics) has to do with *overtones*. A well-made tuning fork or an electronic oscillator emits a pure tone, wholly lacking in overtones. Musical instruments, however, produce tones that are mixtures of pure tones: the fundamental (which determines the pitch we hear) and its overtones. The relative strengths of these pure tones determine the quality or timbre of the sound. The full sound of a fine violin is rich in overtones, while the pure sound of a recorder is weak in overtones.

The strongest overtones in most musical tones are the first two: the octave and the fifth above the octave. The pitch of the next strongest overtone is two octaves above the fundamental, and of the following one, two octaves and a major third above the fundamental.

Experiments show that two *pure* tones played together produce a dissonant sound only if their pitches are just close enough to be distinguishable. Two *musical* tones played together produce a dissonant sound whenever any of their constituent pure tones are just close enough in pitch to be distinguishable. Octaves and fifths played in tune on musical instruments are consonant because the pitches

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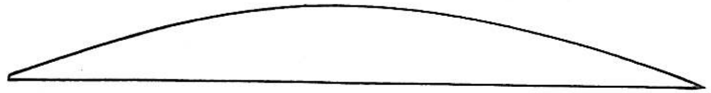


FIGURE 4.1 An ideal string vibrating in its fundamental mode. The waveform is half a sine wave.

of their constituent pure tones either coincide or are well separated. Unisons, octaves, and fifths played slightly out of tune are especially dissonant because their strongest pure constituents are close but distinguishable in pitch.

The fundamental and its overtones correspond to different ways (modes) in which an ideal string with fixed endpoints can vibrate. When an ideal string vibrates in the fundamental mode its waveform is half a sine wave (Figure 4.1). The waveform of the first overtone, whose pitch is an octave higher than that of the fundamental, is a full sine wave (Figure 4.2). The midpoint of the string is a node of the vibration; that is, it remains at rest. You can make a bowed string vibrate in this mode by touching it lightly at this point. The waveform of the next overtone has two nodes (Figure 4.3).

In each of these modes, every point on the string except the endpoints and the nodes (if there are any) moves up and down periodically. If we plot the vertical displacement of a single point on the string against time, we get another sine curve (Figure 4.4). The curves that represent the vertical motion at different points differ only in amplitude. This kind of vibration is called *harmonic*, because a sound wave in which the air pressure varies in this way carries a pure musical tone.

The duration of a single cycle of a vibration is called the period. The periods of the various modes are simply related. The period of the first overtone is one-half the period of the fundamental; the period of the second overtone is one-third the period of the fundamental; and so on. Thus the period of a mode is proportional to its wavelength.

Up until now, we've discussed *possible* modes of vibration of a perfect violin string. In practice, however, a plucked or bowed violin string never vibrates in one of these modes. Plucking or bowing excites a *composite mode*—a *superposition* of simple harmonic modes, each vibrating with its own period as though the other modes weren't there. The meaning of this statement is illustrated in Figure 4.5. To construct the waveform of a composite mode, we add the displacements

FIGURE 4.2 An ideal string vibrating in its first overtone. The waveform is a full sine wave.

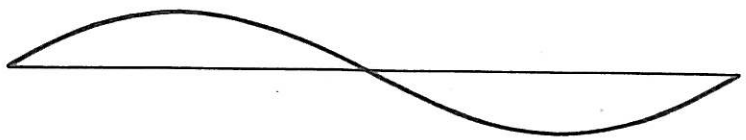


FIGURE 4.3 An ideal string vibrating in its second overtone. The waveform is a sine wave and a half.

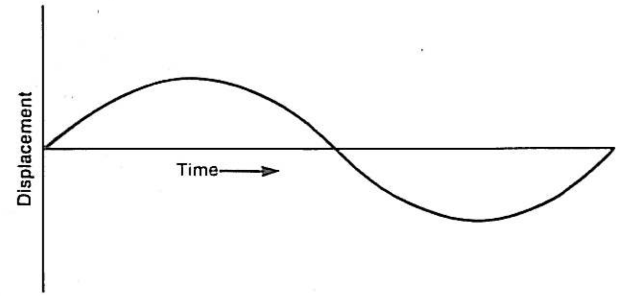
associated with its constituent simple harmonic modes, treating upward displacements from the string's unperturbed position as positive quantities and downward displacements as negative quantities, so that equal and opposite displacements cancel.

The contribution of each simple harmonic mode to the superposition is proportional to the mode's *amplitude*. The amplitude of a mode is a positive or negative number equal in magnitude to the height of the highest point of the waveform. If the amplitude is positive, that mode's contributions to the net displacement at each point are added to the contributions of the remaining modes; if it is negative, they are subtracted. Thus in Figure 4.5, the amplitudes of two modes (the fundamental and the fourth overtone) are positive, and the amplitude of the third mode (the second overtone) is negative. To specify a composite mode, we have to specify the amplitude of each of its constituent harmonic modes.

The vibrating string exemplifies two important properties shared by all vibrating systems.

1. *Simple harmonic modes.* Every vibrating system has simple harmonic modes, in which the displacement at every point is represented, as in Figure 4.4, by a sine curve with a fixed period. In general, the ratios between the periods of the simple harmonic modes are not whole numbers, as they are for the vibrating string. And the waveforms of the simple harmonic modes have less simple shapes

FIGURE 4.4 The vertical displacement of a single point on a vibrating string plotted against time. The plotted points lie on a sine curve. The width of the curve represents the period of the vibration.



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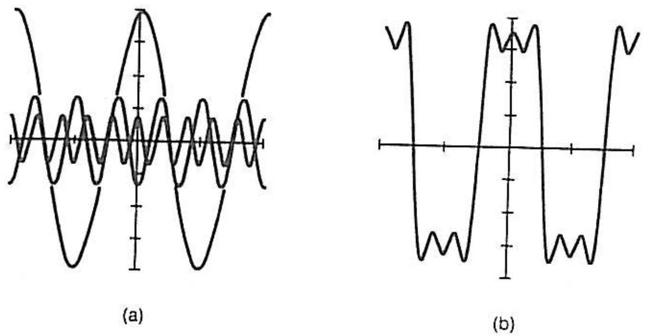

FIGURE 4.5 Constructing a composite mode by adding simple modes. (a) Three sinusoidal waves whose wavelengths are in the ratios 1:3:5. (b) The composite wave formed from these simple waves. The three waves interfere constructively at the center of the illustrated region and interfere destructively on either side of this region. (From *The Science of Musical Sound*, by John R. Pierce. Copyright © 1983 Scientific American Books Inc. Reprinted with permission)

than those of a vibrating string. Figure 4.6 shows the waveforms of some harmonic modes of a vibrating drumhead of uniform thickness. The modes illustrated here are all symmetric about the drumhead's axis of symmetry, a line perpendicular to the drumhead through its center. Although the periods of the overtones are not submultiples (1/2, 1/3, 1/4, and so on) of the period of the fundamental, as they are for the vibrating string, they do get smaller as the number of nodes in the waveform gets larger.

2. *Superposition.* A vibrating system can vibrate in several modes at once, each mode vibrating as though the others weren't there. The displacement at any point is the algebraic sum of the displacements resulting from the individual modes—upward displacements being counted as positive, negative displacements as negative.

Physical States of the Hydrogen Atom

Now consider a hydrogen atom, which consists of a single electron bound by electrostatic attraction to a proton (a particle about 2,000 times as massive as an electron, carrying 1 unit of positive electric charge). Because the proton is so massive, we may treat it as a fixed center of attraction, just as we treat the Sun as a fixed center of attraction for the planets. If the electron behaved like a planet revolving around the Sun, its possible physical states would be represented by orbits. According to quantum physics, however, electrons bound in atoms don't behave like planets.

The possible physical states of a hydrogen atom are analogous to the modes of vibration of a vibrating system. States in which the energy of the hydrogen

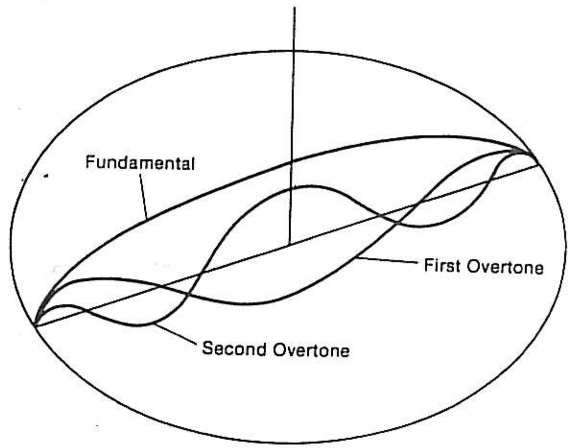

FIGURE 4.6 Vertical cross section of a drumhead of uniform thickness, virbrating in its three lowest axially symmetric modes (schematic). The waveforms are *not* sine curves, and the periods of the first and second overtones are not submultiples of the period of the fundamental. The frequency (reciprocal period) of the first overtone is 2.3 times the frequency of the fundamental (instead of 2 times the frequency of the fundamental, as it is for a vibrating string). The frequency of the second overtone is 3.6 times (instead of 3 times) the frequency of the fundamental.

atom has a definite value correspond to simple harmonic vibrations (which have a definite period). The most general state of a hydrogen atom corresponds to a superposition of simple harmonic vibrations.

The vibrations that represent physical states of a hydrogen atom are three dimensional. Figure 4.7 shows the waveforms of some harmonic modes in which the displacement has the same value at every point on any given sphere centered on the nucleus. These vibrations are analogous to the pulsations of a rubber ball that has been compressed and released. (Certain stars whose brightness varies periodically pulsate in the same way.)

As in vibrating strings and drumheads, the harmonic vibration with the longest period—the fundamental—has a nodeless waveform, and the period of the vibration decreases as the number of nodes increases. The periods are given by a remarkably simple rule. Let P_1 stand for the period of the fundamental, P_2 for the period of the first spherically symmetric overtone, and so on. Then the ratio between any two periods is equal to the ratio of the squares of the corresponding whole numbers. For example, the period of the first overtone, P_2, is 2^2, or 4, times the period of the fundamental, P_1; the period of the second overtone, P_3, is 3^2, or 9, times the period of the fundamental; and so on. This is the connection I hinted at earlier between the discreteness of the physical states of a hydrogen atom and the Pythagorean theory of musical harmony. I think the Pythagoreans would

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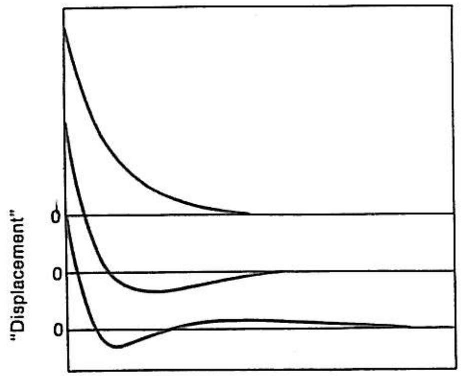

FIGURE 4.7 Waveforms of three spherically symmetric "vibrations" that represent physical states of the hydrogen atom. "Displacement" is plotted against distance from the nucleus.

have found it appropriate that the kinds of vibrations that produce pure musical tones also underlie the structure of hydrogen, the most abundant constituent of the Universe, and that the pitches of these vibrations—audible only to the mind's ear, like the music of the heavenly spheres—should be connected to the integers by a simple rule.

If we accept the analogy between states of a hydrogen atom and vibrations of a vibrating system, we can understand why the structure of a hydrogen atom, unlike the structure of a planetary system, is immune to change. Waveforms, like the spheres and cubes of Euclidean geometry, are mathematical objects, endowed with permanent and immutable properties. We can disrupt a hydrogen atom, but we can't alter the waveforms and pitches of the simple vibrations that represent its possible states; and when the proton and the electron of the disrupted atom recombine to form a new atom, it will be indistinguishable from every other hydrogen atom in the Universe. But what is the basis for the analogy? In what way do the physical states of a hydrogen atom resemble vibrations? How are the waveforms and periods of these vibrations connected with quantities that physicists can actually measure?

The best way to understand the quantum physicist's picture of a hydrogen atom is to retrace the route that led to it. We will see how and why physicists were forced to jettison their cherished belief in the possibility of picturing atomic and subatomic structures and processes. Classical physics had populated the world with two distinct classes of energy-carrying objects: waves and particles. A wave is in many places at the same time; a particle is always in a particular place. Light consists of waves; matter, of particles. Quantum physics fused these two concepts, endowing light with particlelike properties and material particles with wavelike properties. The quantum picture of physical reality is not a picture at all in the conventional sense. We can grasp it, but we can't visualize it.

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